On the S–wave πD–scattering length in the relativistic field theory model of the deuteron

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Abstract

The S–wave scattering length of the strong pion–deuteron (πD) scattering is calculated in the relativistic field theory model of the deuteron suggested in [1,2].The theoretical result agrees well with the experimental data. The important role of the Δ–resonance contribution to the elastic πD–scattering is confirmed.

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1 Introduction

In our previous publications [1,2] we have suggested a relativistic field theory model of the deuteron. The basis of the model is the one–nucleon loop origin of a physical deuteron produced by low–energy fluctuations of the proton and neutron. Therefore, the model can be applied to the description of only low–energy interactions of the deuteron. All interactions of a physical deuteron with other particles should be determined only by one–nucleon loops [1,2]. This recipe is very similar to the approximation accepted within the Nambu–Jona–Lasinio model, where all interactions of hadrons, collective quark–antiquark excitations, are obtained through one–constituent quark–loop exchanges [3-7]. Also one has to understand that most likely the deuteron cannot be inserted in an intermediate state of any process of low–energy interactions. This is connected with a very sensitive structure of the deuteron as an "extended" bound state with a small binding energy. The representation of such a state in terms of any local quantum field is rather limited. The latter entails an undetermined character of the description of the deuteron in intermediate states in terms of Green functions of these local fields.

In this paper we apply such a model to the computation of the S-wave elastic πD–scattering.

The effective Lagrangian of the physical deuteron field $D_{\mu}(x)$ interacting strongly with the proton $p(x)$, neutron $n(x)$ and pion $\vec{\pi}(x)$ fields reads [1,2]

$$\begin{aligned}\mathcal{L}_{\rm st}(x) &= -\frac{1}{2}D^{\dagger}_{\mu\nu}(x)D^{\mu\nu}(x) + M^2_{\rm D}D^{\dagger}_{\mu}(x)D^{\mu}(x) + \\ &+ g_{\rm V}\bar{N}(x)\gamma^{\mu}\tau^2 N^c(x)D_{\mu}(x) + g_{\rm V}\bar{N^c}(x)\tau^2\gamma^{\mu}N(x)D^{\dagger}_{\mu}(x) + \\ &+ \mathcal{L}_{\pi \rm NN}(x)\,,\end{aligned}\qquad (1.1)$$

where $\vec{\tau} = (\tau^1, \tau^2, \tau^3)$ are Pauli matrices of the isotopical spin, and $N(x)$ is the nucleon field, the isotopical doublet with components $N(x) = (p(x), n(x))$. Under isotopical rotations the field $\tau_2 N^c(x)$ transforms like $N(x)$. Then $D_{\mu\nu}(x) = \partial_{\mu}D_{\nu}(x) - \partial_{\nu}D_{\mu}(x)$, $M_{\rm D} = 2M_{\rm N} - \varepsilon_{\rm D}$ is the mass of the physical deuteron, where $M_{\rm N} = 938\,{\rm MeV}$ is the mass of the proton and neutron and $\varepsilon_{\rm D} = 2.225\,{\rm MeV}$ is the binding energy of the deuteron [8], $g_{\rm V}$ is the phenomenological coupling constant of the model [2], that can be expressed in terms of the electric quadrupole moment of the deuteron $Q_{\rm D}$ [2], i.e., $g^2_{\rm V}/\pi^2 = Q_{\rm D}M^2_{\rm D}/2$. As $Q_{\rm D} = 0.286\,{\rm fm}^2$ [8] we get $g_{\rm V} = 11.3$ [2]. Then $\psi^c(x) = C\bar{\psi}^T(x)$ and $\bar{\psi}^c(x) = \psi^T(x)C$, where the indices c and T imply a charge conjugation and a transposition, and $C = i\gamma^2\gamma^0$ is the matrix of a charge conjugation.

The effective Lagrangian $\mathcal{L}_{\pi \rm NN}(x)$ describes strong low–energy interactions of neucleons and pions. Since it is well–known that chiral $SU(2)\times SU(2)$ symmetry plays an important role for strong low–energy interactions of pions and nucleons, we should use the pion–nucleon interactions invariant under chiral $SU(2)\times SU(2)$ transformations.

There are two possibilities for the realization of chiral $SU(2)\times SU(2)$ symmetry, i.e. using either linear or nonlinear representation. In a linear realization of chiral $SU(2)\times SU(2)$ symmetry one needs to introduce a scalar isoscalar field $\sigma(x)$ with the mass of the order $M_{\sigma} \sim 700\,{\rm MeV}$ being a partner of a pion field under chiral rotations [4-7]. Unfortunately, up to now the scalar isoscalar meson with the mass $M_{\sigma} \sim 700\,{\rm MeV}$ was not observed experimentally. In turn, within a nonlinear realization of chiral $SU(2)\times SU(2)$ symmetry, a pion field transforms nonlinearly under chiral rotations and the scalar

isoscalar field $\sigma(x)$ does not appear. To avoid the problem of dealing with unobserved state we suggest to use a nonlinear realization of chiral $SU(2)\times SU(2)$ symmetry. In this case, following Weinberg [9], the Lagrangian $\mathcal{L}_{\pi\mathrm{NN}}(x)$ should read

$$\begin{aligned}
&\mathcal{L}_{\pi\mathrm{NN}}(x) = \bar{N}(x)(i\gamma^{\mu}\partial_{\mu} - M_{\mathrm{N}})N(x) - \left[1 + \frac{\vec{\pi}^{\,2}(x)}{4F_0^2}\right]^{-1}\times\\
&\left[\frac{g_{\pi\mathrm{NN}}}{2M_{\mathrm{N}}}\bar{N}(x)\gamma^{\mu}\gamma^5\vec{\tau}N(x)\cdot\partial_{\mu}\vec{\pi}(x) + \frac{1}{4F_0^2}\,\bar{N}(x)\gamma^{\mu}\vec{\tau}N(x)\cdot(\vec{\pi}(x)\times\partial_{\mu}\vec{\pi}(x))\right]\\
&+\frac{1}{2}\partial_{\mu}\vec{\pi}(x)\cdot\partial^{\mu}\vec{\pi}(x)\left[1 + \frac{\vec{\pi}^{\,2}(x)}{4F_0^2}\right]^{-1} - \frac{1}{2}M_{\pi}^2\vec{\pi}^{\,2}(x)\left[1 + \frac{\vec{\pi}^{\,2}(x)}{4F_0^2}\right]^{-1} = \\
&= \bar{N}(x)(i\gamma^{\mu}\partial_{\mu} - M_{\mathrm{N}})N(x) + \frac{1}{2}\partial_{\mu}\vec{\pi}(x)\cdot\partial^{\mu}\vec{\pi}(x) - \frac{1}{2}M_{\pi}^2\vec{\pi}^{\,2}(x) - \\
&-\frac{g_{\pi\mathrm{NN}}}{2M_{\mathrm{N}}}\bar{N}(x)\gamma^{\mu}\gamma^5\vec{\tau}N(x)\cdot\partial_{\mu}\vec{\pi}(x) - \frac{1}{4F_0^2}\,\bar{N}(x)\gamma^{\mu}\vec{\tau}N(x)\cdot(\vec{\pi}(x)\times\partial_{\mu}\vec{\pi}(x))\\
&+\ldots,
\end{aligned}\tag{1.2}$$

where $M_{\pi} = M_{\pi^0} = 134.976\,\mathrm{MeV}$ is the mass of the pion field $\vec{\pi}(x)$, $F_0 \simeq 92\,\mathrm{MeV}$ is the PCAC constant of π–mesons calculated in the chiral limit [7], and $g_{\pi\mathrm{NN}}$ is the πNN–coupling constant. Below we set $g_{\pi\mathrm{NN}} = 13.4 \pm 0.2$ [2,10].

However, it is well–known [11] that the πNN–interaction is not sufficient to explain strong low–energy interactions of the deuteron, and the interaction of pions and nucleons with the Δ–resonance, i.e. the $\pi\mathrm{N}\Delta$–interaction, plays an important role. Following [12], we take the $\pi\mathrm{N}\Delta$–interaction in the form

$$\mathcal{L}_{\pi\mathrm{N}\Delta}(x) = \frac{g_{\pi\mathrm{N}\Delta}}{2M_N}\,\bar{\Delta}^a_{\mu}(x)\,N(x)\,\partial^{\mu}\pi^a(x) + \mathrm{h.c.},\tag{1.3}$$

where $\Delta^a_{\mu}(x)$ is the Δ–resonance field, the index a runs over $a = 1,2,3$ and

$$\begin{aligned}
\Delta^1 = \tfrac{1}{\sqrt{2}}\begin{pmatrix}\Delta^{++} - \Delta^0/\sqrt{3}\\ \Delta^{+}/\sqrt{3} - \Delta^{-}\end{pmatrix},\ \Delta^2 = \tfrac{i}{\sqrt{2}}\begin{pmatrix}\Delta^{++} + \Delta^0/\sqrt{3}\\ \Delta^{+}/\sqrt{3} + \Delta^{-}\end{pmatrix},\\
\Delta^3 = -\sqrt{\tfrac{2}{3}}\begin{pmatrix}\Delta^{+}\\ \Delta^{0}\end{pmatrix}.
\end{aligned}\tag{1.4}$$

Using the $\pi\mathrm{N}\Delta$–interaction (1.3) we derive the effective Lagrangian describing the πN $\to \Delta \to \pi$N transition

$$\begin{aligned}
&\int d^4x\mathcal{L}^{\pi\mathrm{N}\to\Delta\to\pi\mathrm{N}}_{\mathrm{eff}}(x) = \frac{ig^2_{\pi\mathrm{N}\Delta}}{4M_N^2}\int d^4x_1 d^4x_2\ \times\\
&\times\Big[\bar{N}(x_1) < 0|\mathrm{T}\Big(\Delta^a_{\mu}(x_1)\bar{\Delta}^b_{\nu}(x_2)\Big)|0> N(x_2)\Big]\,\partial^{\mu}\pi^a(x_1)\partial^{\nu}\pi^b(x_2),
\end{aligned}\tag{1.5}$$

where

$$< 0|\mathrm{T}\Big(\Delta^a_{\mu}(x_1)\bar{\Delta}^b_{\nu}(x_2)\Big)|0 >= -i\left(\frac{2}{3}\delta^{ab} - \frac{1}{3}i\varepsilon^{abc}\tau^c\right)S_{\mu\nu}(x_1 - x_2).\tag{1.6}$$

We define the momentum representation of $S_{\mu\nu}(x)$ in accordance with [13]

$$S_{\mu\nu}(p) = \frac{1}{M_{\Delta} - \hat{p}}\left(-\,g_{\mu\nu} + \frac{1}{3}\gamma_{\mu}\gamma_{\nu} + \frac{1}{3}\,\frac{\gamma_{\mu}p_{\nu} - \gamma_{\nu}p_{\mu}}{M_{\Delta}} + \frac{2}{3}\,\frac{p_{\mu}p_{\nu}}{M^2_{\Delta}}\right),\tag{1.7}$$

where $M_\Delta = 1232$ MeV is the Δ–resonance mass.

Substituting (1.6) in (1.5) we get

$$\int d^4x \mathcal{L}_{\rm eff}^{\pi{\rm N}\to\Delta\to\pi{\rm N}}(x) = \tag{1.8}$$
$$= \frac{g^2_{\pi{\rm N}\Delta}}{6M^2_N}\int d^4x_1 d^4x_2 \times \Big[\bar{N}(x_1)S_{\mu\nu}(x_1 - x_2)N(x_2)\Big]\,\partial^\mu\;\vec{\pi}(x_1)\cdot\partial^\nu\vec{\pi}(x_2) -$$
$$-\frac{g^2_{\pi{\rm N}\Delta}}{12M^2_N}\int d^4x_1 d^4x_2\Big[\bar{N}(x_1)S_{\mu\nu}(x_1 - x_2)i\vec{\tau}\,N(x_2)\Big]\cdot(\partial^\mu\vec{\pi}(x_1)\times\partial^\nu\vec{\pi}(x_2)).$$

The coupling constant $g_{\pi{\rm N}\Delta}$ is connected with $g_{\pi{\rm NN}}$ via the relation $g_{\pi{\rm N}\Delta} = 1.90\,g_{\pi{\rm NN}}$ [12,14].

We should emphasize that we do not take into account the $\pi\Delta\Delta$–interaction. It is because in our model defined in one–baryon loop approximation the contribution of the $\pi\Delta\Delta$–interaction is the matter of two–baryon loop approximation.

2 Elastic πD–scattering

The effective Lagrangian $\mathcal{L}_{\rm eff}^{\pi{\rm D}\to\pi{\rm D}}(x)$, describing elastic πD–scattering at low energies, contains two contributions

$$\mathcal{L}_{\rm eff}^{\pi{\rm D}\to\pi{\rm D}}(x) = \mathcal{L}_{\rm eff}^{\pi{\rm D}\to{\rm NN}\to\pi{\rm D}}(x) + \mathcal{L}_{\rm eff}^{\pi{\rm D}\to{\rm N}\Delta\to\pi{\rm D}}(x), \tag{2.1}$$

where $\mathcal{L}_{\rm eff}^{\pi{\rm D}\to{\rm NN}\to\pi{\rm D}}(x)$ and $\mathcal{L}_{\rm eff}^{\pi{\rm D}\to{\rm N}\Delta\to\pi{\rm D}}(x)$ are represented by the simple box–diagrams and defined

$$\begin{aligned}&\int d^4x\,\mathcal{L}_{\rm eff}^{\pi{\rm D}\to{\rm NN}\to\pi{\rm D}}(x)\;=\int d^4x\int\frac{d^4x_1\,d^4k_1}{(2\pi)^4}\,\frac{d^4x_2\,d^4k_2}{(2\pi)^4}\,\frac{d^4x_3\,d^4k_3}{(2\pi)^4}\times\\&\times D_\mu(x)\,D^\dagger_\nu(x_1)\,\partial_\alpha\vec{\pi}(x_2)\cdot\partial_\beta\vec{\pi}(x_3)\times\\&\times e^{-ik_1\cdot x_1}e^{-ik_2\cdot x_2}e^{-ik_3\cdot x_3}e^{i(k_1+k_2+k_3)\cdot x}\,\frac{g^2_{\rm V}}{8\pi^2}\frac{g^2_{\pi{\rm NN}}}{4M^2_{\rm N}}\,\mathcal{J}^{\mu\nu\alpha\beta}(k_1,k_2,k_3;Q)_{\rm NN}\,,\end{aligned}\tag{2.2}$$

$$\begin{aligned}&\int d^4x\,\tilde{\mathcal{L}}_{\rm eff}^{\pi{\rm D}\to{\rm N}\Delta\to\pi{\rm D}}(x)\;=\int d^4x\int\frac{d^4x_1\,d^4k_1}{(2\pi)^4}\,\frac{d^4x_2\,d^4k_2}{(2\pi)^4}\,\frac{d^4x_3\,d^4k_3}{(2\pi)^4}\times\\&\times D_\mu(x)\,D^\dagger_\nu(x_1)\,\partial_\alpha\vec{\pi}(x_2)\cdot\partial_\beta\vec{\pi}(x_3)\times\\&\times e^{-ik_1\cdot x_1}e^{-ik_2\cdot x_2}e^{-ik_3\cdot x_3}e^{i(k_1+k_2+k_3)\cdot x}\,\frac{g^2_{\rm V}}{8\pi^2}\frac{g^2_{\pi{\rm N}\Delta}}{6M^2_{\rm N}}\mathcal{J}^{\mu\nu\alpha\beta}(k_1,k_2,k_3;Q)_{{\rm N}\Delta}.\end{aligned}\tag{2.3}$$

The structure functions $\mathcal{J}^{\mu\nu\alpha\beta}(k_1,k_2,k_3;Q)_{\rm NN}$ and $\mathcal{J}^{\mu\nu\alpha\beta}(k_1,k_2,k_3;Q)_{{\rm N}\Delta}$ are given by

$$\begin{aligned}&\mathcal{J}^{\mu\nu\alpha\beta}(k_1,k_2,k_3;Q)_{\rm NN} = \int\frac{d^4k}{\pi^2\,i}{\rm tr}\,\{S_{\rm F}(k+Q)\gamma^\mu S_{\rm F}(k+Q+k_1)\gamma^\nu\\&S_{\rm F}(k+Q+k_1+k_2)\gamma^\alpha\gamma^5 S_{\rm F}(k+Q+k_1+k_2+k_3)\gamma^\beta\gamma^5\}\,,\\&\mathcal{J}^{\mu\nu\alpha\beta}(k_1,k_2,k_3;Q)_{{\rm N}\Delta} = \int\frac{d^4k}{\pi^2\,i}{\rm tr}\,\{S_{\rm F}(k+Q)\gamma^\mu S_{\rm F}(k+Q+k_1)\gamma^\nu\\&S_{\rm F}(k+Q+k_1+k_2)S^{\alpha\beta}(k+Q+k_1+k_2+k_3)\}\,,\end{aligned}\tag{2.4}$$

where $S_{\rm F}(p)$ is a Green function of a free nucleon in the momentum representation

$$S_{\rm F}(p) = \frac{1}{M_{\rm N} - \hat{p}}\,. \tag{2.5}$$

Then $Q = a\,k_1 + b\,k_2 + c\,k_3$ is an arbitrary shift of a virtual momentum. Fortunately, the result of the computation of the integrals (2.4) taken in leading long–wavelength approximation [1,2] does not depend on the shift of a virtual momentum and can be computed unambiguously. Holding the terms giving the main contribution in long–wavelength expansion [1,2] we obtain

$$\begin{aligned}
{\cal J}^{\mu\nu\alpha\beta}(k_1,k_2,k_3;Q)_{\rm NN} &=& -\frac{16}{9}\,g^{\mu\nu}\,g^{\alpha\beta} + \ldots,\\
{\cal J}^{\mu\nu\alpha\beta}(k_1,k_2,k_3;Q)_{\rm N\Delta} &=& -\frac{20}{9}\,g^{\mu\nu}\,g^{\alpha\beta} + \ldots.
\end{aligned} \tag{2.6}$$

We have neglected the contribution of the mass difference $M_{\Delta} - M_{\rm N}$ that is small compared with the remained part. Also we have dropped out the divergent contributions that can be expressed in terms of the integrals [1,2]

$$\begin{aligned}
J_1(M_{\rm N}) &=& \int\frac{d^4k}{\pi^2 i}\frac{1}{M^2_{\rm N} - k^2} = 4\int^{\Lambda_{\rm D}}_0\frac{d|\vec{k}|\vec{k}^{\,2}}{(M^2_{\rm N}\,+\,\vec{k}^{\,2})^{\,1/2}}\,,\\
J_2(M_{\rm N}) &=& \int\frac{d^4k}{\pi^2 i}\frac{1}{(M^2_{\rm N} - k^2)^{\,2}} = 2\int^{\Lambda_{\rm D}}_0\frac{d|\vec{k}|\vec{k}^{\,2}}{(M^2_{\rm N}\,+\,\vec{k}^{\,2})^{\,3/2}}\,.
\end{aligned} \tag{2.7}$$

The ultraviolet cut–off has the meaning of the upper limit $\Lambda_{\rm D} = 64.843\,{\rm MeV}$ restricting the 3–momenta of fluctuations of virtual nucleons taking part in the formation of the physical deuteron field [1,2]. In terms of the cut–off $\Lambda_{\rm D} = 64.843$ MeV we define an effective radius of the deuteron $r_{\rm D} = 1/\Lambda_{\rm D} = 3.043$ fm [2]. This value agrees well with the average value of the deuteron radius $\bar{r} = \int d^3x\,r\,|\psi_{\rm D}(\vec{r}\,)|^2 = 3.140$ fm [15], where $\psi_{\rm D}(\vec{r}\,)$ is the wave–function of the deuteron at the ground state. The effective radius $r_{\rm D} = 1/\Lambda_{\rm D} = 3.043$ fm is compared qualitatively with the deuteron radius $r_{\rm D} = (\varepsilon_{\rm D}M_{\rm N})^{-1/2} = 4.319$ fm [8], defined in terms of the binding energy of the deuteron $\varepsilon_{\rm D} = 2.225$ MeV [8]. Due to inequality $M_{\rm N} \gg \Lambda_{\rm D}$ the term depending on the cut–off $\Lambda_{\rm D}$ is small compared with convergent contributions.

Applying structure functions Eq.(2.6) we compute the effective Lagrangian describing the elastic low–energy πD–scattering

$${\cal L}^{\pi\rm D\to\pi D}_{\rm eff}(x) \;=\; -\frac{1}{9}\,Q_{\rm D}\left(g^2_{\pi\rm NN} + \frac{5}{6}\,g^2_{\pi\rm N\Delta}\right)D^{\dagger}_{\mu}(x)\,D^{\mu}(x)\,\partial_{\nu}\vec{\pi}(x)\cdot\partial^{\nu}\vec{\pi}(x). \tag{2.8}$$

The amplitude of the elastic low–energy πD–scattering defined by the effective Lagrangian (2.6) reads

$${\cal A}(s,t,u) \;=\; \frac{1}{9}\,Q_{\rm D}\left(g^2_{\pi\rm NN} + \frac{5}{6}\,g^2_{\pi\rm N\Delta}\right)(t - 2M^2_{\pi})\,. \tag{2.9}$$

In turn the S–wave length of the elastic πD–scattering is given by

$$
\begin{aligned}
a_{\pi\mathrm{D}} &= \frac{1}{8\pi}\frac{1}{M_{\mathrm{D}}+M_\pi}\mathcal{A}(s,t,u)\Big|_{\mathrm{threshold}} = \\
&= -\frac{1}{36\pi}\left(g^2_{\pi\mathrm{NN}}+\frac{5}{6}\,g^2_{\pi\mathrm{N}\Delta}\right)\frac{Q_{\mathrm{D}}M^2_\pi}{M_{\mathrm{D}}+M_\pi} = -\,0.057\,M^{-1}_\pi\,.
\end{aligned}
\tag{2.10}
$$

The S–wave scattering length Eq.(2.10) is of order $O(M^2_\pi)$. This agrees with the results obtained by Robilotta [16] and Weinberg [17]. The theoretical magnitude of $a_{\pi\mathrm{D}}$ given by Eq.(2.10) is reasonably well compared with experimental data [18]

$$
(a_{\pi\mathrm{D}})_{\mathrm{exp}} = -0.052^{+0.022}_{-0.017}\,M^{-1}_\pi. \tag{2.11}
$$

One can see that the $\pi\mathrm{N}\Delta$–interaction gives the main contribution to the low–energy S–wave elastic πD–scattering. This confirms the important role of the Δ–resonance for strong low–energy πD–dynamics.

3 Conclusion

We have shown that the relativistic field theory model of the deuteron supplemented by the effective pion–nucleon interaction derived within nonlinear realization of chiral $SU(2)\times SU(2)$ symmetry and contribution of the Δ–resonance describes well the dynamics of low–energy elastic πD–scattering at energies very close to the πD–threshold.

The magnitude of the S–wave scattering length is found to be well compared with experimental data due to the inclusion of the $\pi\mathrm{N}\Delta$–interaction. This confirms a well–known assertion concerning an important role of the Δ–resonance in the elastic πD–scattering.

Thus we can conclude that the relativistic field theory model of the deuteron really can be applied to the description of strong low–energy πD–dynamics at energies very close to the πD–threshold.

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